

STRUCTURE OF THE CAUCHY HORIZON SINGULARITY

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We study the Cauchy horizon (CH) singularity of a spherical charged black hole perturbed nonlinearly by a self-gravitating massless scalar field. We show numerically that the singularity is weak both at the early and at the late sections of the CH, where the focusing of the area coordinate r is strong. In the early section the metric perturbations vanish, and the fields behave according to perturbation analysis. We find exact analytical expressions for the gradients of r and of the scalar field, which are valid at both sections. We then verify these analytical results numerically.

1 Introduction

In the last few years, there has been a steadily growing evidence that a new type of singularity forms at the Cauchy horizon (CH) of spinning or charged black holes (BHs). The features of this new singularity differ drastically from those of the previously-known singularities like, e.g., Schwarzschild or BKL: First, the CH singularity is null rather than spacelike; Second, it is weak ¹. Namely, the tidal distortion experienced by an infalling extended test body is finite (and, moreover, is typically negligibly small) as it hits the singularity. Yet, curvature scalars diverge there ².

Despite recent advances ^{3,4,5,6,7}, our understanding of the null weak CH singularity is still far from being complete. In particular, it is important to verify this new picture by performing independent, non-perturbative, analyses. This motivates one to employ numerical tools to study the structure of the CH singularity. The numerical simulation of spinning BHs is difficult, as they are non-spherical. One is thus led to study, numerically, the inner structure of a spherical charged BH; hopefully, it may serve as a useful toy model for a spinning BH.

Recently, Brady and Smith (BS) ⁸ numerically explored the mass-inflation singularity inside a spherical charged BH perturbed by a spherical scalar field. This analysis confirmed several aspects of the above new picture: It demonstrated the existence of a null singularity at the CH, where the mass function m diverges but the area coordinate r is nonzero. r was found to decrease monotonically, due to the nonlinear focusing, until it shrinks to zero (at which point the singularity becomes spacelike). It also provided evidence for the weakness of the singularity. Despite its remarkable achievements, however, this analysis left one important issue unresolved: To what extent is the perturbative approach applicable at (and near) the CH singularity? BS reported on an inconsistency with the predictions of perturbation analysis, manifested by the non-zero value of σ (see ⁸). This issue is crucial, because for realistic (i.e., spinning and uncharged) BHs the only direct evidence at present for the actual occurrence of a null weak singularity stems from the perturbative analysis ³. A failure of the perturbative approach in the spherical charged case would therefore undermine the confidence in our understanding of realistic

BHs' interiors. In the next section we briefly describe our numerical and analytical results. Further details on this research will appear elsewhere ⁹.

2 The numerical and analytical investigations

We consider the model of a spherical charged BH perturbed non-linearly by a spherical, self-gravitating, neutral, massless scalar field Φ . Our numerical code is based on free evolution of the dynamical equations in double-null coordinates ^{10,11}. The code is stable and second-order accurate. Our initial-value setup is described in Ref. ¹⁰: The geometry is initially Reissner-Nordström (RN), with initial mass $M_0 = 1$ and charge Q , and no scalar field. At some retarded moment v , however, it is modified by an ingoing scalar-field pulse of a squared-sine shape with amplitude A . Φ vanishes everywhere on the initial surface except in a finite range $v_1 < v < v_2$. In this case, due to the scalar-field energy, the BH's external mass approaches the final mass M_f . Note that our outgoing initial null hypersurface is located outside the event horizon (EH) (unlike in Ref. ⁸).

Our numerical simulations confirm the presence of a null singularity at the CH, where m diverges and r is nonzero. Along the CH singularity, r decreases monotonically, until it shrinks to zero, at which point the singularity becomes spacelike. This situation was already found numerically by BS ⁸. We next investigate the early part of the CH singularity, i.e., the part where the focusing of r is still negligible. Our first goal is to demonstrate that the singularity is weak. In terms of the double-null metric, the singularity will be weak if coordinates $\hat{u}(u), \hat{v}(v)$ can be chosen such that both r and $g_{\hat{u}\hat{v}}$ are finite and nonzero at the CH. The numerical analysis by BS already demonstrated the finiteness of r , which we recover in our results. (Note that r is independent of the choice of the null coordinates.) Figure 1(A) displays the metric function $g \equiv -2g_{\hat{u}\hat{v}}$ in Kruskal-like coordinates U, V along an outgoing null ray that intersects the early section of the CH singularity. The CH is located at $V = 0$ (corresponding to $v \rightarrow \infty$). This figure clearly demonstrates the finiteness of g_{UV} , from which the weakness of the singularity follows. The perturbation analysis also predicts that both the scalar field and the metric perturbations will be arbitrarily small at the early section of the CH. In other words, both metric functions r and g should be arbitrarily close to the corresponding RN metric functions. This behavior is indeed demonstrated in Fig. 1. In this figure, g and r are displayed along lines $u = \text{const}$ [Figs. 1(A) and 1(B)] and $v = \text{const}$ [Figs. 1(C) and 1(D)]. The RN values are denoted by solid lines, and the numerical values in circles. The similarity of the analytic RN functions and the numerically-obtained functions of the perturbed spacetime is remarkable. (We emphasize, though, that despite the similarity in the values of the metric functions to RN, our geometry is drastically different from that of RN, in the sense that in our case curvature blows up at the CH, as manifested by the rapid growth of m [Fig. 1(E)].)

We turn now to explore the late part of the CH singularity, where focusing is strong. First, we numerically verify the weakness of the singularity in this part too. Figure 1(F) shows $K \equiv -2g_{uV}$ along an outgoing null ray in the late part of the CH, where the focusing level (of r) is approximately 90%. We present here the results for various values of the grid-parameter N (see ¹⁰), in order to demonstrate the second-

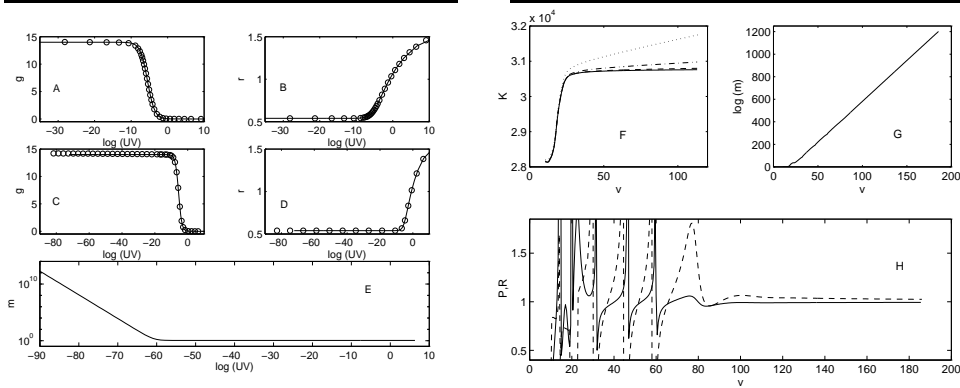


Figure 1: Right: The metric functions and the mass function at the early parts of the CH. Left: metric, mass function and normalized blue-shift factors at the late parts of the CH.

order numerical convergence. g_{uV} approaches a finite value at the CH ($v \rightarrow \infty$). At the same time, the mass function (and curvature) grows exponentially with v [Fig. 1(G)]. We therefore conclude that the entire null CH singularity is weak, even at the region of strong focusing.

Next we study, analytically, the behavior of the blue-shift factors $r_{,v}$ and $\Phi_{,v}$ along the contracting CH. The field equations can be integrated exactly along the CH singularity. We find $R \equiv \Psi_{,v}/\{\Psi_{,v}^{\text{EH}}[2(M_f/Q)^2 - 1]\} = 1$ and $P \equiv -(r^2)_{,v_e}/[(1/\kappa_-)\Psi_{,v_e}^2] = 1$. Here, κ_- is the surface gravity at the RN inner horizon with parameters M_f and Q , and $\Psi \equiv r\Phi$. Note that P is invariant to a gauge transformation $v \rightarrow \tilde{v}(v)$, whereas R is not. In order to verify these expressions, we calculated numerically and plotted R (dashed) and P (solid), as functions of v , along an outgoing ray located at a region of 90% focusing of the CH. The results, presented in Fig. 1(H), are in excellent agreement with the above theoretical prediction, $R = 1$ and $P = 1$.

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